O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 Linear Systems
Fall 2000
Final Exam

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Problem 1:
In the circuit shown below, $v_i(t)$ is a voltage source and nonlinear resistor obeys the relation $i_R = 1.5v_R^3$. $v_i(t)$ is the circuit input and $v_R(t)$ is the circuit output. Derive the differential equation for this circuit. Linearize this differential equation for the case when the circuit operates about the point $v_i = 14$.

Problem 2:
Let l_i be an eigenvalue of a matrix A and let v^i be the corresponding eigenvector. Let $f(l) = \sum_{k=0}^{l} a_k l^k$ be a polynomial with real coefficients a_k. Show that $f(l_i)$ is an eigenvalue of the matrix function $f(A) = \sum_{k=0}^{l} a_k A^k$ with the same coefficients a_k. Determine the eigenvector corresponding to eigenvalue $f(l_i)$.

Problem 3:

For the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix},$$

determine the functions of matrix A, $e^{\cos At}$.

Problem 4:

Find an "equivalent" continuous-time *Jordan-canonical-form* dynamical equation of

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ -1 & 1 \end{bmatrix} u(k),$$

$$y(k) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}.$$

Problem 5:

Show that if all eigenvalues of A are distinct, then $(sI - A)^{-1}$ can be expressed as

$$(sI - A)^{-1} = \sum_i \frac{1}{s - \lambda_i} q_i p_i$$

where q_i and p_i are right and left eigenvectors of A associated with λ_i.

Note: q_i is a *right* eigenvector of A associated with λ_i, $Aq_i = \lambda_i q_i$ and p_i is a *left* eigenvector of A associated with λ_i, $p_i A = \lambda_i p_i$.